ST. XAVIER'S SENIOR SECONDARY SCHOOL, DELHI – 110 054

Std. 11 **Time : 3 hrs.**

24-2-2024 **Final Examination in MATHEMATICS** **Max. Marks : 80**

General Instructions:

i) This question paper consists of 38 questions divided into four sections A, B, C, D and E. All questions are compulsory.

ii) Section A consists of 18 M.C.Q and 02 assertion reason based questions carrying 1 mark each. Section B consists of 5 very short answer questions carrying 2 marks each. Section C consists of 6 short answer questions carrying 3 marks each. Section D consists of 4 long answer questions carrying 5 marks each. Section E has 3 source based/case based questions carrying 4 marks each.

iii) There is no overall choice. However, an internal choice has been given in some questions.

SECTION – A

1. $8\sin\frac{x}{8}\cos\frac{x}{2}\cos\frac{x}{4}\cos\frac{x}{8}$ is equal to:
   a) $8cosx$ b) $cosx$ c) $8sinx$ d) $sinx$

2. If $A = \{x: x\ is\ a\ multiple\ of 3\}$ and $B = \{x: x\ is\ a\ multiple\ of\ 5\}$ then $A - B$ is equal to:
   a) $A \cap B$ b) $A \cap B'$ c) $A' \cap B'$ d) $(A \cap B)'$

3. A line passes through the point (2,2) is perpendicular to the line $3x + y = 3$. Its y- intercept is:
   a) 4/3 b) 2/3 c) 1 d) -1

4. Let $A = \{x: x \in R, x \geq 4\}$ and $B = \{x: x \in R, x < 5\}$ then $A \cap B$ is:
   a) [4,5] b) (4,5) c) [4,5) d) (4,5]

5. If $A + B = \frac{\pi}{2}$, then $\frac{sinA+sinB}{cosA-cosB}$ is equal to:
   a) 1 b) 2 c) 0 d) -1

6. The range of $f(x) = \sqrt{x-1}$ is:
   a) (0, ∞) b) (-∞, 1] c) [0, ∞) d) [1, ∞)

7. Let A and B are two events and if P(A) = 0.42 and P(B) = 0.48 and P(A and B) = 0.16, then P(not B) is:
   a) 0.32 b) 0.46 c) 0.52 d) 0.58

8. If $20_{c_r} = 20_{c_{r-10}}$, then the value of r is:
   a) 12 b) 15 c) 20 d) 10

9. $\lim_{x\to 2} \frac{x^3-8}{x^2-4}$ is equal to:
   a) 3 b) 2 c) 4 d) 1

10. The solution set of inequation $\frac{2}{x-4} > 0$ is:
    a) [4, ∞] b) [-4, ∞) c) $(-\infty, 4)$ d) $(4, \infty)$

11. The radius of a circle $2x^2 + 2y^2 - x = 0$ is equal to:
    a) 4 b) 1/4 c) 1/16 d) 2

12. An unbiased coin is tossed thrice. If the third toss gets head, what is the probability of getting at least one more head:
    a) 1/4 b) 1/2 c) 3/4 d) 1/3

13. If $y = \frac{\sin(x+a)}{cosx}$, then $\frac{dy}{dx}$ at $x = 0$ is equal to:
a) $cosa$ b) 1 c) 0 d) $sina$

14. The distance of a point (3,5) from the line $3x - 4y + 26 = 0$ is
a) 5 unit b) 3 unit c) 1 unit d) 4 unit

15. The length of latus rectum of ellipse $\frac{x^2}{25} + \frac{y^2}{100} = 1$ is
a) 10 unit b) 5 unit c) 7 unit d) 12 unit

16. The point (-2, 1, -3) lies in:
a) II octant b) III octant c) VI octant d) IV octant

17. If $y = \sqrt{x} + \frac{1}{\sqrt{x}}$, then $\frac{dy}{dx}$ at $x = 1$ is:
a) 1 b) ½ c) 2 d) 0

18. The value of $\lim_{x \to \frac{\pi}{2}} \frac{1-\sin x}{\cos x}$ is:
a) 1 b) 0 c) 2 d) 4

Questions 19 and 20, a statement of assertion (A) is followed by a statement of reason (R). Choose the correct answer from the given choices.

19. Assertion (A): If $(a + 3, b - 2) = (3\ , 1)$, then $a = 2$ and $b = 3$.
Reason (R): Two ordered pairs are equal, if their corresponding elements are equal.
a) Both A and R is true and R is correct explanation of A.
b) Both A and R is true but R is not a correct explanation of A.
c) A is true but R is false.
d) A is false but R is true.

20. Assertion (A): The value of 99^3 is equal to 970299.
Reason (R): $(1 + x)^n = 1 + n_{c_1}x + n_{c_2}x^2 + \cdots \ldots \ldots \ldots n_{c_n}x^n$.
a) Both A and R is true and R is correct explanation of A.
b) Both A and R is true but R is not a correct explanation of A.
c) A is true but R is false.
d) A is false but R is true.

SECTION – B

21. Let $U = \{x: x \in n, x \leq 9\}$; $A = \{x: x\ is\ an\ even\ number, 0 < x < 10\}$; $B = \{2, 3, 5, 7\}$.
Write the set $(A \cup B)^{/}$.

22. If (a, b) is the midpoint of the line segment between coordinate axes, show that the equation of the line is $\frac{x}{a} + \frac{y}{b} = 2$.

23. Evaluate: $\lim_{x \to 0} \frac{\sqrt{x+1}-1}{x}$.

(OR)

Evaluate: $\lim_{x \to a} \frac{x^9 - a^9}{x - a} = 9$, find all possible values of a.

24. Find the equation of set of all points P such that its distance from the points A (3, 4, -5) and B (-2, 1, 4) are equal.

25. Solve the system of inequalities and represent the solution graphically on number line.
$3x - 7 < 5 + x \; and \; 11 - 5x \leq 1$

(OR)

Find all pair of consecutive odd natural numbers, both of which are larger than 10, such that their sum is less than 40.

SECTION – C

26. If $y = \left(\sin\frac{x}{2} + \cos\frac{x}{2}\right)^2$, then find $\frac{dy}{dx}$ at $x = \frac{\pi}{3}$.

27. Evaluate$(\sqrt{3} + \sqrt{2})^6 - \left(\sqrt{3} - \sqrt{2}\right)^6$.

(OR)

Using binomial theorem, prove that $6^n - 5n$ always leaves remainder 1 when divided by 25.

28. Prove that $cos^2x + cos^2(x + \frac{\pi}{3}) + cos^2(x - \frac{\pi}{3}) = \frac{3}{2}$.

(OR)

Prove that $\frac{\tan 5x + \tan 3x}{\tan 5x - \tan 3x} = 4\cos 2x \cos 4x$.

29. Find the total number of arrangements that can be formed from the letters of the word INDEPENDENCE. In how many of these arrangements
i) do all the vowels occur together? ii) do all the words begin with P?

(OR)

If $2n + 1_{P_{n-1}} : 2n - 1_{P_n} = 3:5$. Find n.

30. If 4-digit number greater than 5000 is randomly formed from the digits 0, 1, 3, 5 and 7, what is the probability of forming a number divisible by 5 when, the digits are not repeated?

31. Find domain and range for the following real valued functions.
i) $f(x) = \frac{4-x}{x-4}$ ii) $f(x) = \sqrt{x^2 + 1}$

SECTION – D

32. A line is such that its segment between line $5x - y + 4 = 0$ and $3x + 4y - 4 = 0$ is bisected at the point (1,5). Obtain its equation.

(OR)

Find the distance between the parallel lines $9x + 6y - 7 = 0$ and 3x + 2y + 6=0. Also find the equation of line which is equidistant from the given lines.

33. Given, $y = \frac{\sin x - x\cos x}{x\sin x + \cos x}$. Prove that $\frac{dy}{dx} = \frac{x^2}{(x\sin x + \cos x)^2}$.

(OR)

Evaluate $\lim_{x \to 0} \frac{(x+h)\sin(x+h) - xsinx}{h}$.

34. Find mean, variance and standard deviation for the given data.

Class interval	30-40	40-50	50-60	60-70	70-80	80-90	90-100
frequency	3	7	12	15	8	3	2

35. Find the equation of circle which passes through the points (2, 3), (4, 5) and having its center on the line $y - 4x + 3 = 0$.

SECTION – E

36. Indian track and field athlete Neeraj Chopra, who competes in the Javelin throw, won a gold medal at Tokyo Olympics. He is the first track and field athlete to win a gold medal in Olympics.

Using above information find:

i) Name the path followed by Javelin.

ii) If the equation of such path is given by $x^2 = -16y$, find the coordinate of its focus.

iii) Find the equation of parabola whose vertex is (0, 0), passing through (2, 3), and axis along x-axis.

37. 

Ravi and Siddhartha are playing cards. Total number of cards is 52 in numbers. Each of them draw four cards one by one On the basis of this information answer the following questions.

i) What is the probability that all four cards drawn by Ravi are of same suit?

ii) What is the probability that one of the four cards drawn by Siddhartha is an ace?

iii) What is the probability that one card is drawn from each suit by Ravi?

(OR)

What is the probability that cards drawn by Siddhartha are of different number?

38. The three important limits are (i) $\lim_{x\to 0}\frac{sinx}{x} = 1$ (ii) $\lim_{x\to 0}\frac{1-cosx}{x} = 0$ (iii) $\lim_{x\to 0}\frac{tanx}{x} = 1$

Using above information, answer the following questions.

i) Find the value of $\lim_{x\to 0}\frac{\sin 3x}{5x}$.

ii) Find the value of $\lim_{\theta\to b}\frac{\tan(\theta-b)}{\theta-b}$.

iii) Find the value of $\lim_{x\to 0}\frac{\tan 2x-sin2x}{x^3}$. (OR) Find $\lim_{x\to 0}\frac{2\sin x-sin2x}{x^3}$.

-x-x-x-x-x-x-x-x-x-